LEVERAGED ETFs

Back to the Present. . . and Future

SAND KEY
RESEARCH

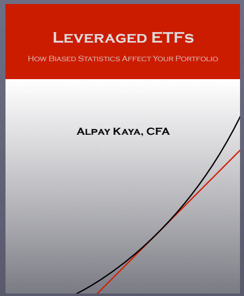
About

Sand Key Research is a research firm specializing in Quantitative Finance. By confronting discrepancies between abstraction and conventional analyses, the firm has developed patent-pending methods for structuring leveraged ETFs and observing market risk.

Sand Key Research was founded by Alpay Kaya, whose book *Leveraged ETFs* derives LETF statistics under 2 return models and presents a theoretically ideal model of leverage. Mr. Kaya is a CFA charterholder and holds Engineering degrees from the University of Akron and UC Berkeley.

Scope

- Introducing a proprietary LETF structure following an overview of leveraged indexing.
- Framework:
  Market → Index → Returns → ETFs
  ↳ Leveraged Returns → LETFs
- Detail: general
  For derivations, see book:
  Leveraged ETFs
  by Alpay Kaya, CFA



Summary

- To The Present
  - Misinformation
  - Misguided Analyses
  - Misguided Solutions
- Future
  - Statistical Analysis
  - Universality
  - LETF 2.0

A STARTING POINT

What We All Know

- Leverage magnitude increases as investors lose and vice versa.

Leverage Exposure/Equity	b(0)	Idx(1)	b(1)
+3x (300/100)	+3	+1%	303/103 = 2.942
	+3	-1%	297/97 = 3.062
-3x (-300/100)	-3	+1%	-303/97 = -3.124
	-3	-1%	-297/103 = -2.883

What We All Know

- Leverage can vary even for a fully-funded position.
  - Consider $|\text{Exposure}| = \text{Equity}$

Leverage Exposure/Equity	b(0)	Idx(1)	b(1)
+1x (300/300)	+1	+1%	303/303 = 1
	+1	-1%	297/297 = 1
-1x (-300/300)	-1	+1%	-303/297 = -1.0202
	-1	-1%	-297/303 = -0.9802

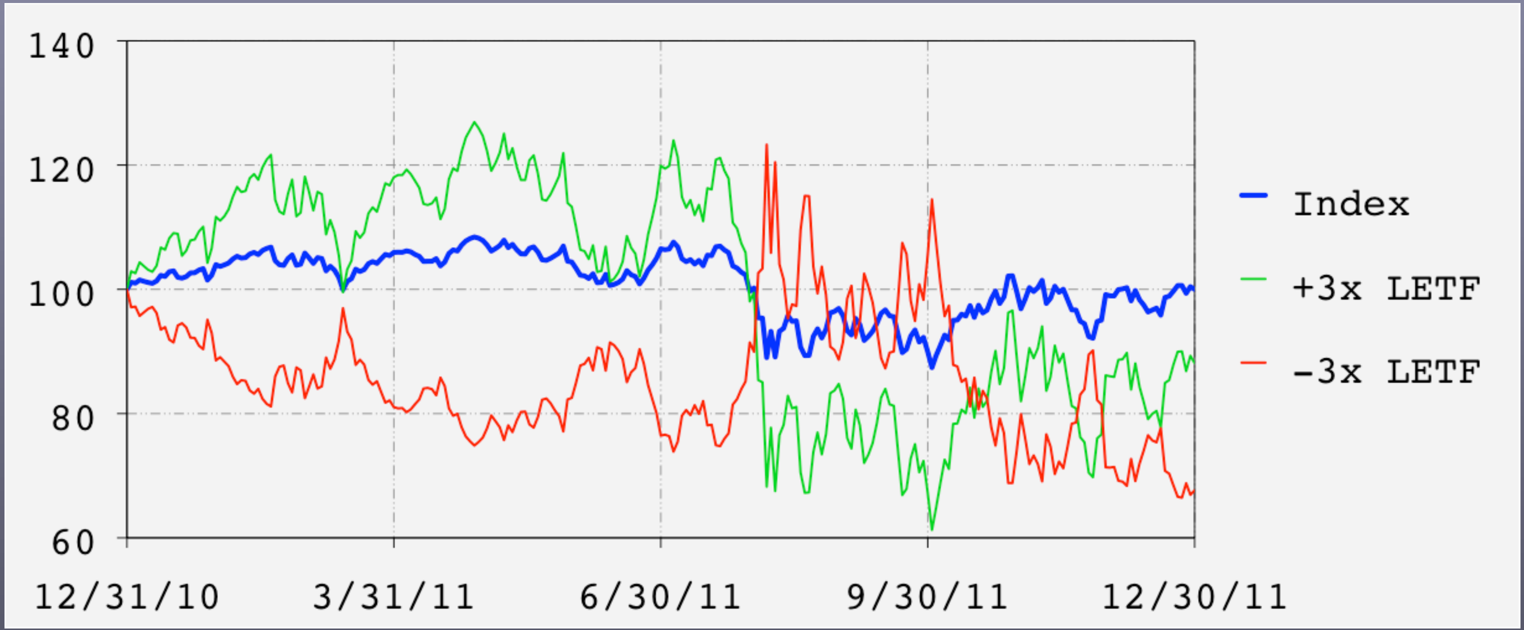
What We All Know

- The common factor among alternative / leveraged ETFs is varying leverage.
- Daily LETF PMs act to reduce the magnitude of leverage as funds lose value, not generally considered to be a high-risk policy.
- ALL short exposures have variable leverage.
  - Increasing rate environment → negative FI exposure → get comfortable with variable leverage

MISINFORMATION

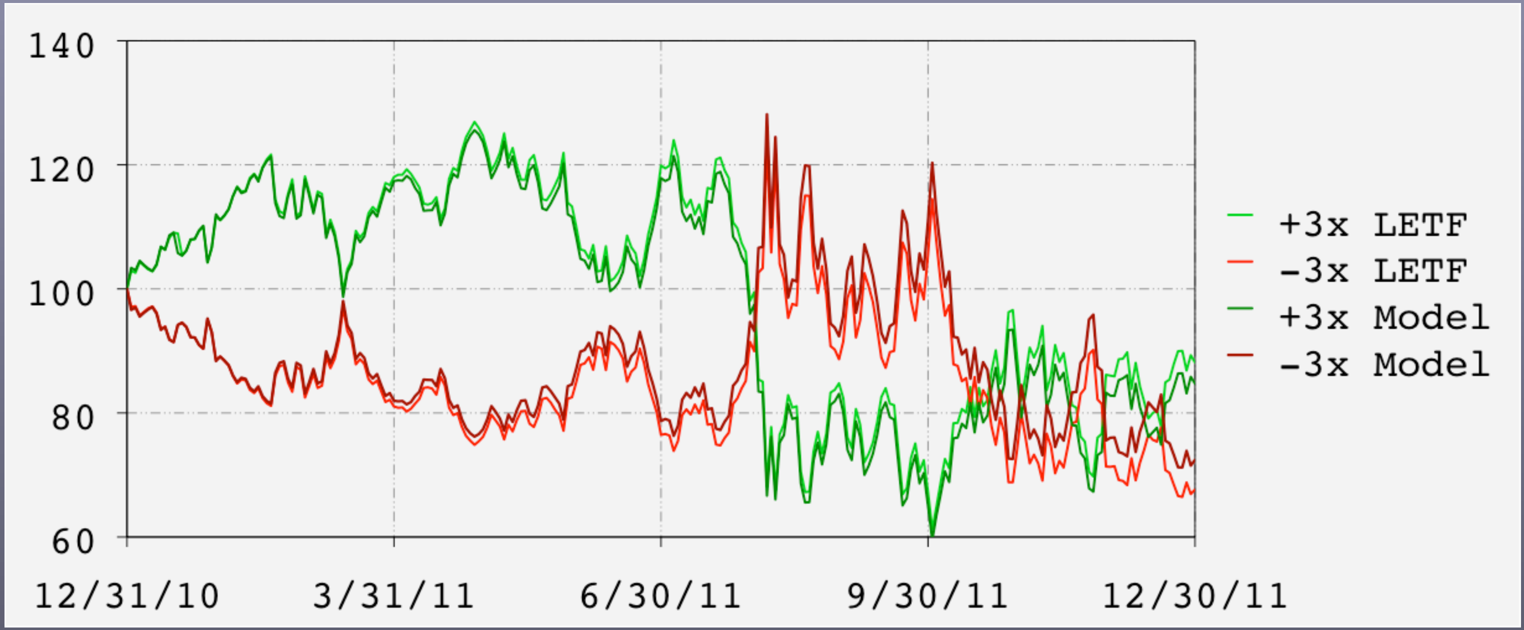
Tracking Error !!!

- The S&P 500 was flat over calendar year 2011.
  - Daily LETFs tracking it were not flat and yet. . .



Tracking Error ???

- Daily LETFs did track an ideal model.



Tracking Error: Conclusion

- The term "tracking error" falsely accuses PMs of incompetence.
  - Model assumptions: zero financing costs, zero management fees, & exact leverage regulation
- Leverage = +1: special case, special properties.
  - Remember: $-1 \neq +1$
- The properties of leverage merit further analysis.

MISGUIDED ANALYSES

1-Step Analysis

- Who needs rebalancing?
  - "All I know is when an index is flat, a passive leveraged portfolio does not lose value!"

A Few Considerations. . .

- This policy is subject to significant leverage swings; for a 15% index move [up, down],
  - `+3x Passive: [ 2.38,  4.64]`
  - `-3x Passive: [-6.27, -1.76]`

1-Step Analysis

Continued,

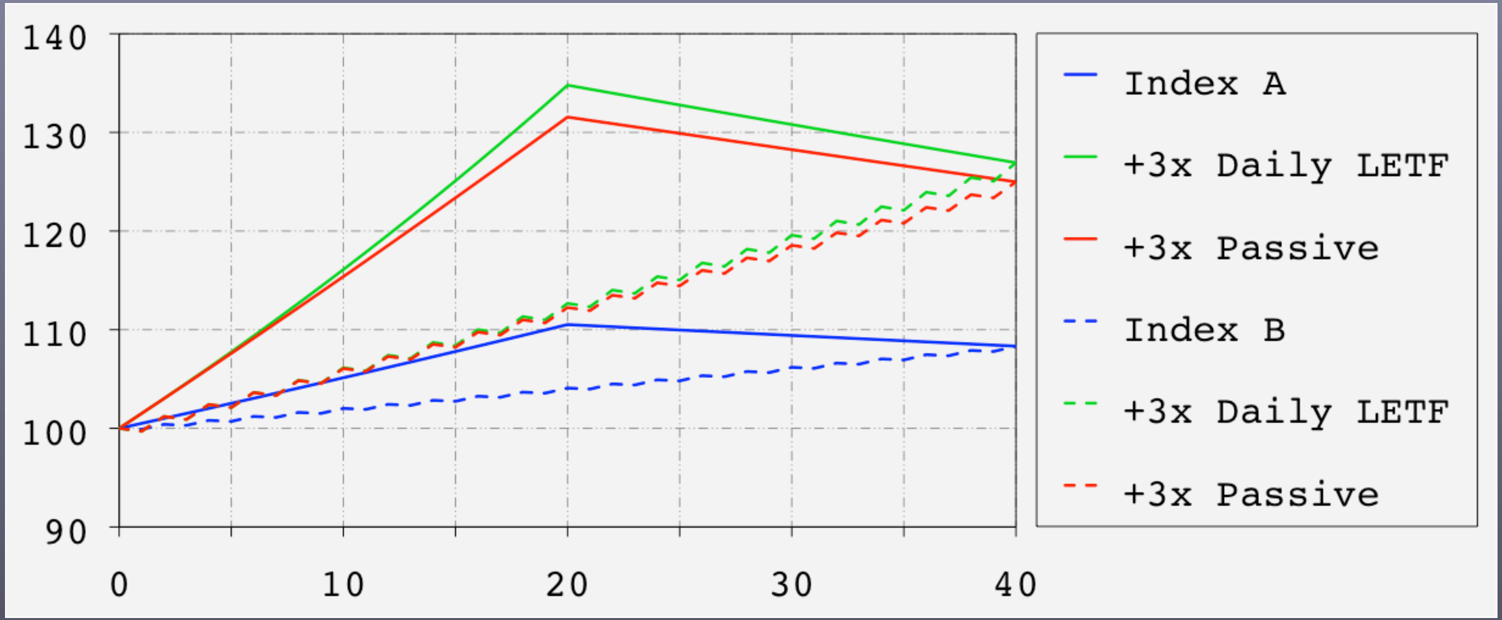
- The possibility of zero equity is real.
  - An index move of $-1/b \rightarrow$ complete loss of equity
  - For +3x / -3x leverage, this means -33% / +33%
- Avoiding zero equity requires rebalancing, making a passive portfolio impractical in the long-term.

2-Step Analysis

- Finance research abounds with 2-step examples attempting to characterize index returns.
  - "Trending" index (up, up)
  - "Flat" index (up, down)
- Factors claimed to relate leveraged returns.
  - Index autocorrelation > 0 $\rightarrow$ Daily LETF > Passive
  - Product of index returns > 0 $\rightarrow$ Daily LETF > Passive
- Such naive analyses are easily proven invalid. . .

2-Step Analysis: Counterexample

- Indices A & B have identical returns reordered.
  - Autocorrelations: Index A = +0.95, Index B = -1
  - Daily LETF & Passive cumulative returns are unaffected



Analysis: Conclusion

- Examples do not a framework make.
- There are analyses well-suited to this topic; serious researchers should make use of them.
- The proper response is NOT a 3-step 'analysis'.

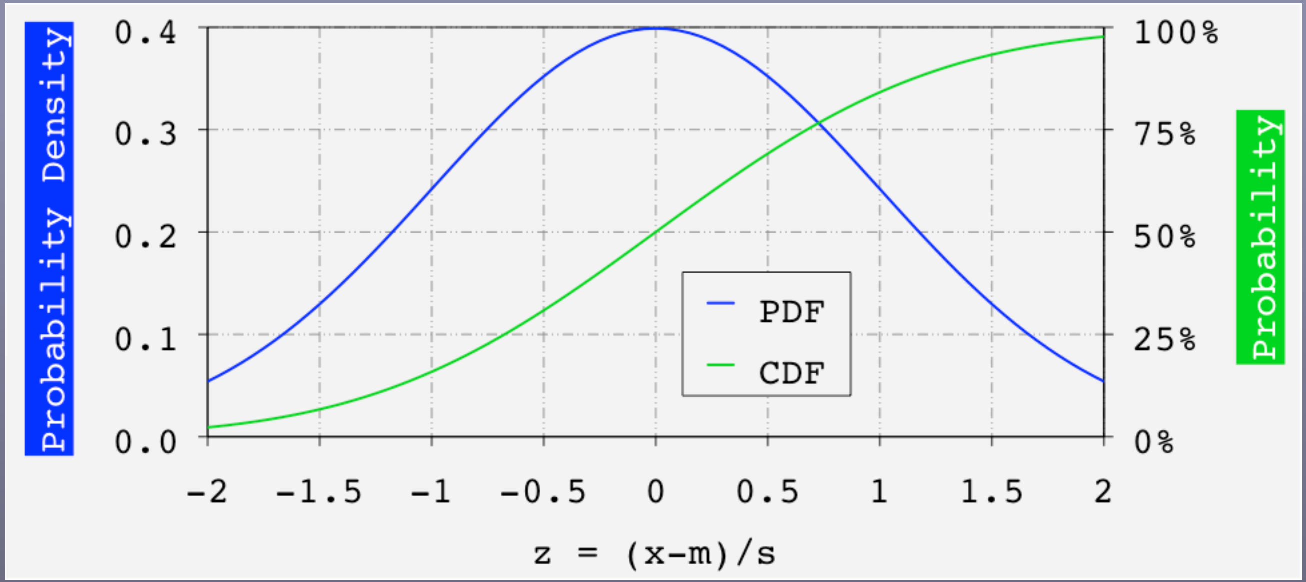
MISGUIDED SOLUTIONS

New/Proposed Products

- Lifetime Fund
  - Imitates a passive portfolio
  - No proposed structure has fungible shares (not an ETF)
- Monthly LETF
  - requires larger rebalancing actions
  - longer-term returns are not genuinely differentiated from Daily LETFs (see upcoming figures)

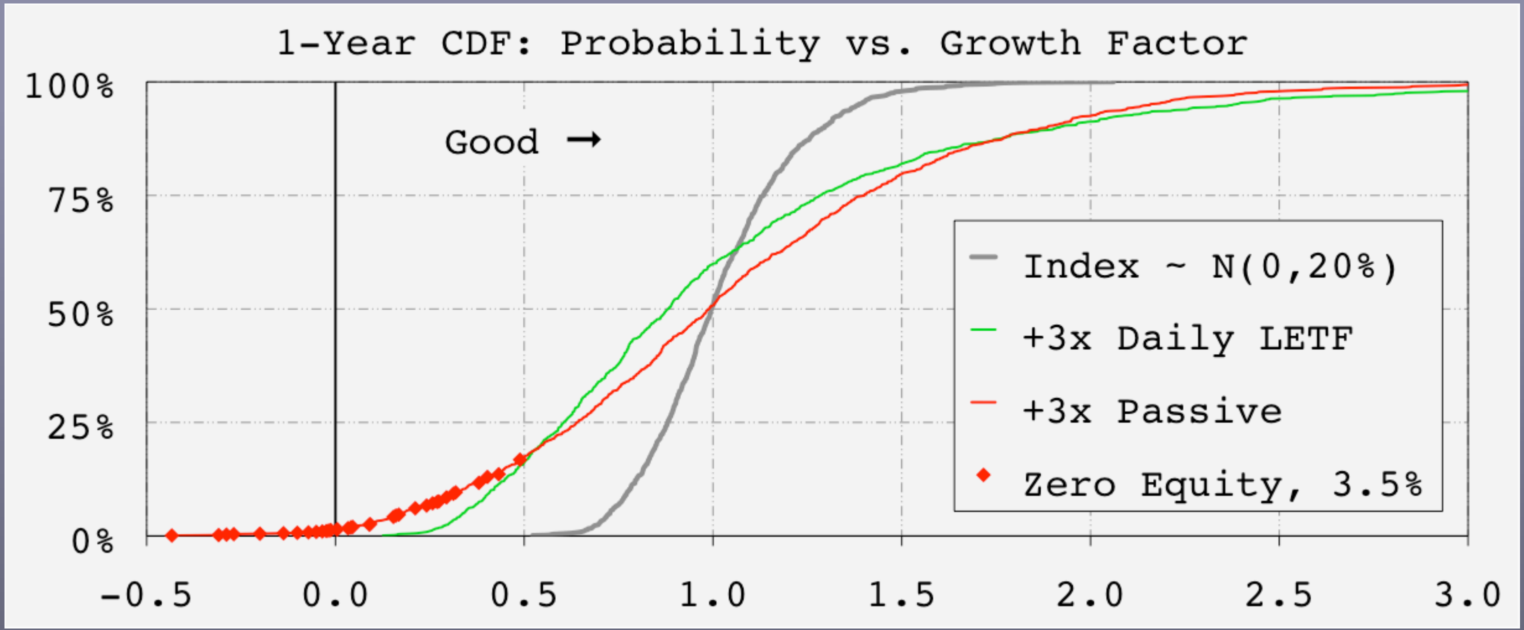
STATISTICAL ANALYSIS

Cumulative Distribution Function



- Why not use a PDF/Histogram?
  - Less noise: a CDF is the integral of a PDF
  - Less subjectivity: a histogram requires choosing buckets

Long-Term: Different Yet The Same



- Expected value doesn't tell the whole story.

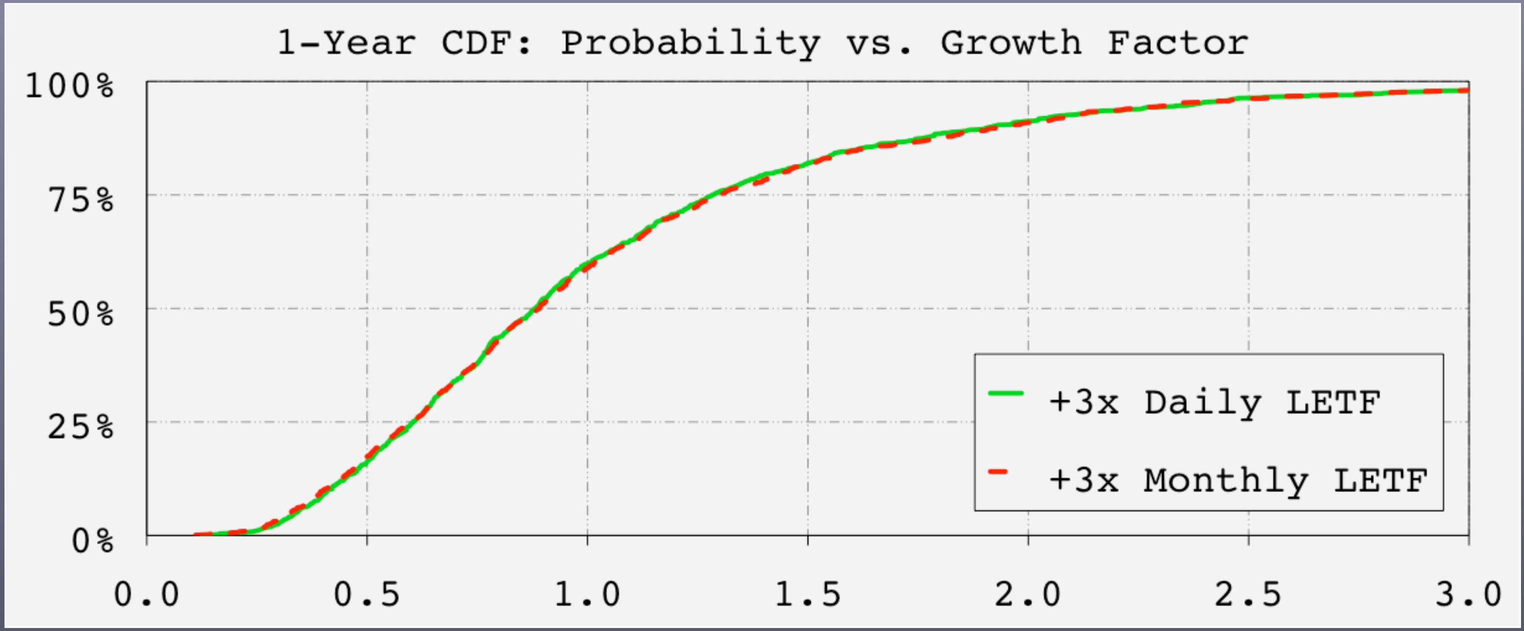
E[Index] = 1.02 → E[LETF] = E[Passive] = 1.06

CDF Returns: Passive vs. Daily LETF

- The difference in median return is about +12%.
- When the index's return is within about 1 standard deviation of its mean, the difference is > 0.
- Passive: zero equity for 3.5% of trajectories.

CDF: Monthly vs. Daily

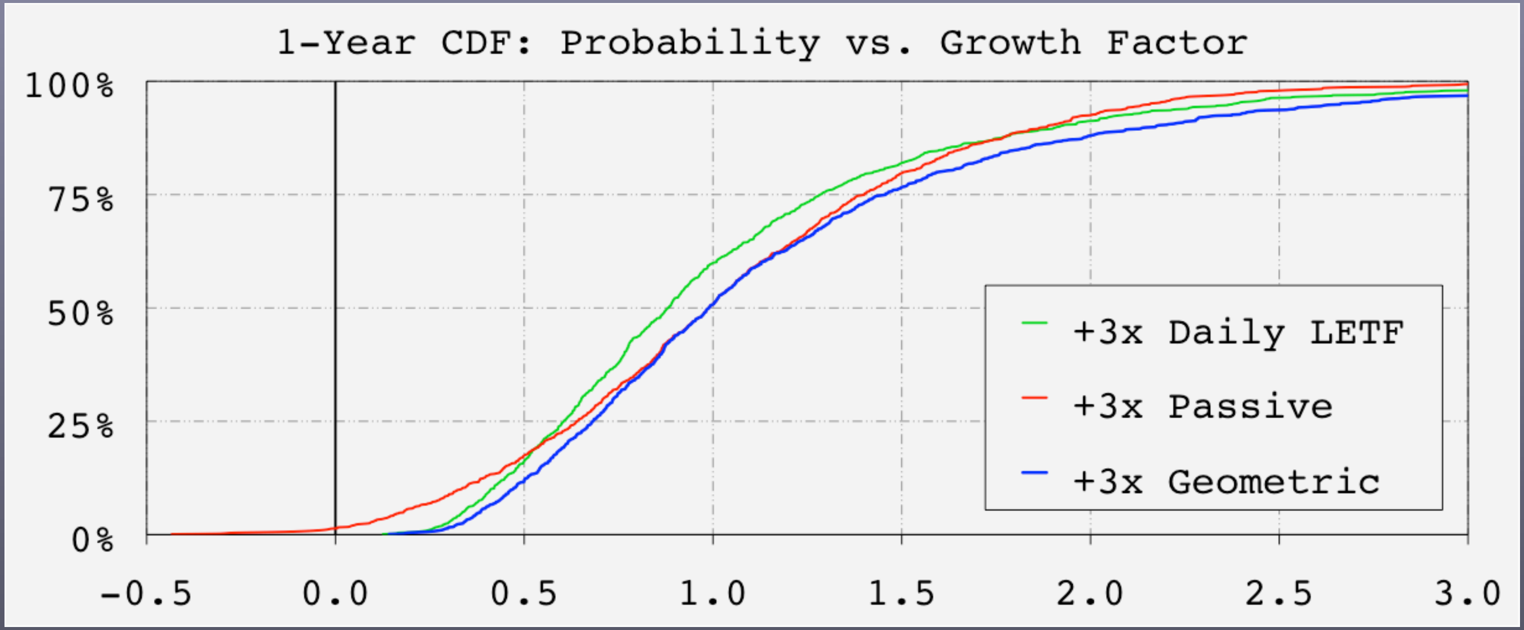
- New class of products offers. . . nothing new?
  - NOT a red & green dashed line



UNIVERSALITY

Leverage Characterization

- Scaling an index's geometric return provides an upper bound (rightmost) CDF.



If It Sounds Too Good. . .

- Geometric leveraged returns are path-independent.
  - Eliminates time interval associated with return objectives
- Future information is required.
  - As far ahead as is needed to effect rebalancing
- A logical analog to 100% energy efficiency.
  - You can get closer, but you can’t reach it

Value Decay Characterization

- Return of a Daily LETF vs. Geometric leverage
  - Generalizes the earlier 1-step analysis in which zero return served as an easily perceivable benchmark
- Decay rate: $\mathbf{r_d = \frac{1}{2}\left(b - b^2\right)s^2}$
- Long vs. Short equivalence: $\mathbf{r_d\left(-|b|\right) = r_d\left(|b| + 1\right)}$
  - Decay rate of a -2x LETF equals that of a +3x LETF
  - Why do -3x LETFs outnumber +4x LETFs?

Index Characterization

- Is an index smooth or volatile over some period?
  - Generalizes the earlier 2-step analysis in which flat was an example of volatile and trending the opposite of flat
- Determining factor:

$$\left|\text{Mean Growth Rate}\right| \Big/ \sqrt{\text{Variance Rate}}$$

- Daily LETF vs. Passive returns
  - Crossover point: ratio > 1 $\rightarrow$ Daily LETF > Passive

Fund Characterization

- There is no universal description of leveraged funds.
- LETF
  - Target leverage & rebalancing period
  - -2x Daily, +3x Monthly, etc.
- Lifetime Fund (Passive)
  - Initial leverage
  - -2x (& no rebalancing), +3x (& no rebalancing), etc.

Fund Characterization

- A universal description of leveraged funds would serve as a framework for R&D.
  - [redacted]
  - Paradigm for new method:

[redacted]

Universality: Conclusion

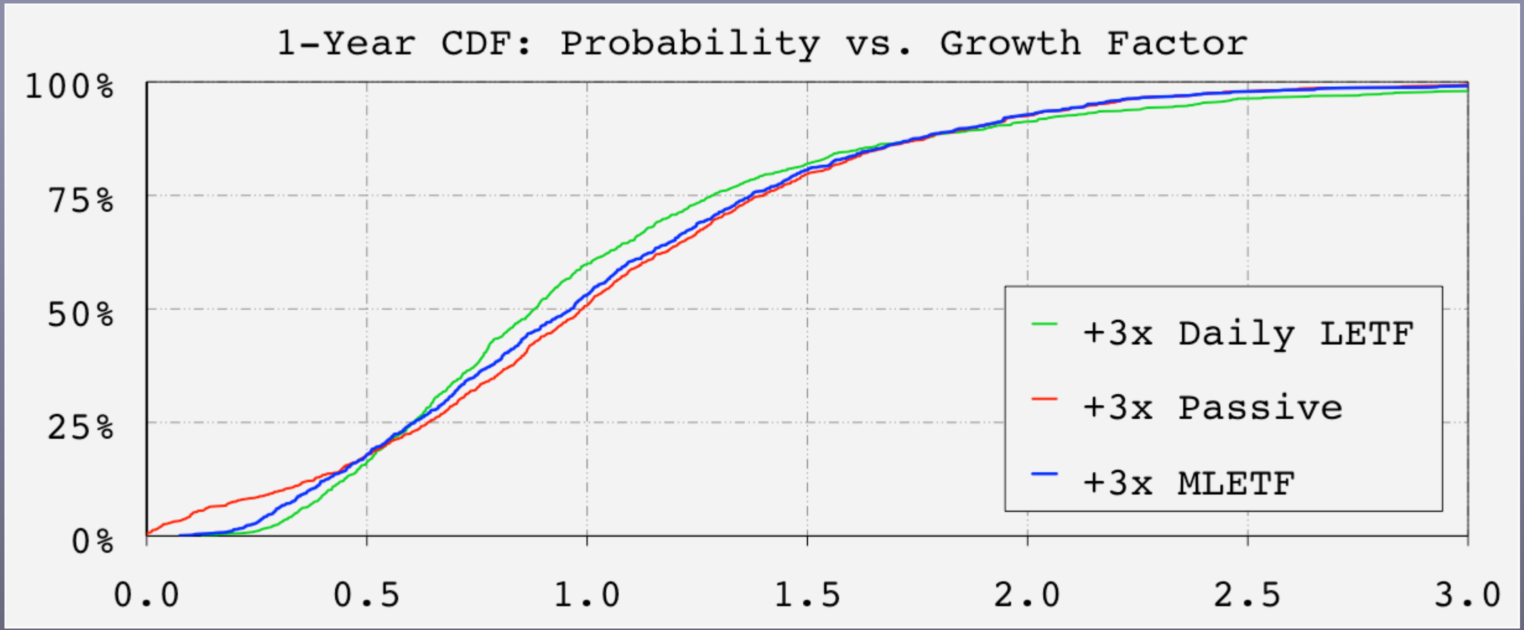
- LETFs generalize conventional financial math.
  - LETF equations simplify when leverage is set to +1
  - Decades of experience with ETFs (leverage = +1) left many unprepared for the characteristics of LETFs
- Product development requires a comprehensive knowledge base.
  - Ad hoc R&D focuses on individual examples, creating one problem as another is 'solved'

LETF 2.0

Multifactorial LETF

- Patent-pending method for structuring LETFs.
- MLETF sponsors may offer investors any mathematically possible CDF.
- MLETF CDFs cannot be replicated by fixed-weight portfolios of preexistent products.
- MLETFs are fungible.

+3x MLETF: a Simple Embodiment

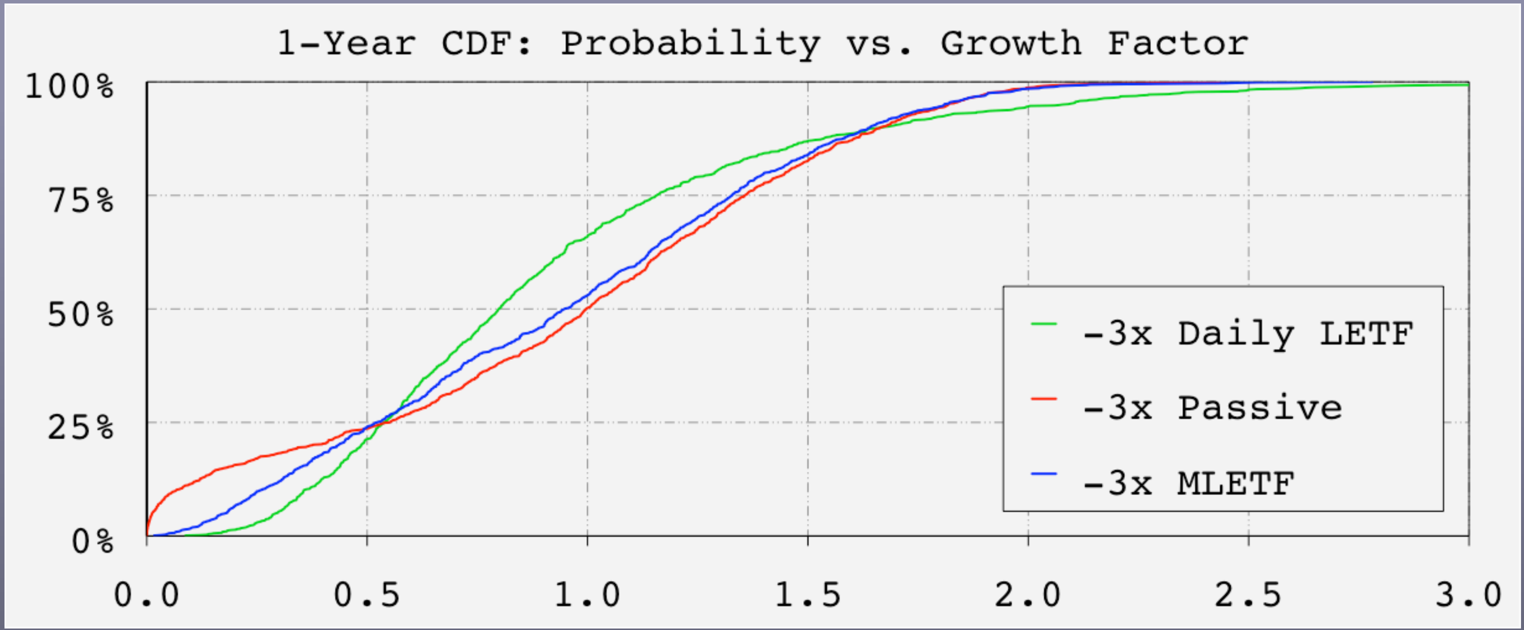


- +3x Passive employs a lenient rebalancing protocol to avoid zero equity.

+3x MLETF: Summary Comparison

	Daily	Monthly	Passive	MLETF
RETURN				
Median	0.882	0.891	0.990	0.965
Mean	1.058	1.059	1.055	1.053
REBALANCING AVERAGES				
Leverage Adjustment	0.06	0.29	5.8	0.10
Interval, Days	1	21	476	29
Annual Lev. Adj.	15.2	3.45	3.06	0.84

-3x MLETF: Same Simple Embodiment



- The -3x MLETF embodiment here is equivalent to that for the +3x MLETF.

-3x MLETF: Summary Comparison

	Daily	Monthly	Passive	MLETF
RETURN				
Median	0.800	0.813	0.947	0.949
Mean	0.929	0.931	0.997	0.950
REBALANCING AVERAGES				
Leverage Adjustment	0.12	0.60	5.3	0.29
Interval, Days	1	21	107	30
Annual Lev. Adj.	30.5	7.24	12.5	2.44

MLETF: Conclusion

- Satisfies the statistical preferences of investors.
  - Provides an advantageous profile across statistical regimes (smooth & volatile) not just in special cases
- Reacts intelligently to index price action.
  - Modest rebalancing actions support market integrity
- Fungible structure supports multiple time horizons.
  - From short-term hedging to long-term investing

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